

where σ_0 is the yield stress at the onset of plastic strain, b is the slope, and $\bar{\epsilon}$ is the effective strain in the plastic region. An inherent implication here is that the wafer material be rigid until the incipience of plastic strain, and then strains in a linear fashion. Thus, the small elastic strains occurring in the wafer are neglected. The effective strain $\bar{\epsilon}$ for a material that is rigid up to yield is given in Reference (f), and when combined with equation (4) can be written as

$$\bar{\epsilon} = \frac{\sqrt{2}}{3} \left[(\epsilon_r - \epsilon_\theta)^2 + (\epsilon_\theta - \epsilon_z)^2 + (\epsilon_z - \epsilon_r)^2 + \frac{3}{2} \gamma_{rz}^2 \right]^{1/2} \quad (10)$$

The von Mises yield theory is thus a combination of equations (8), (9), and (10).

The two equilibrium equations for cylindrical coordinates are easily developed from the stress state acting on a differential volume element taken from the wafer in the loaded state. They are

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_0}{r} + \frac{\partial \tau_{rz}}{\partial z} = 0 \quad (11)$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0 \quad (12)$$